FP3 Mock Kerime 2

1. $4\tanh^2 x - 2\operatorname{sech}^2 x = 3$

$\therefore\ 4 - 4\operatorname{sech}^2 x - 2\operatorname{sech}^2 x = 3$

$\therefore\ 4 - 6\operatorname{sech}^2 x = 3$

$6\operatorname{sech}^2 x = 1$

$\Rightarrow \operatorname{sech}^2 x = 1/6$

$\Rightarrow \cosh^2 x = 6$

$\cosh x > 0$

$\Rightarrow \cosh x = \sqrt{6}$

$\Rightarrow \operatorname{arcosh}\sqrt{6} = \ln\left(\sqrt{6} + \sqrt{(6+1)}\right)$

$\cosh(x) \equiv \cosh(\pm x) \Rightarrow x = \pm \ln\left(\sqrt{6} + \sqrt{5}\right)$

2. $y = 2\cosh\left(\frac{1}{2}x\right)$ $\qquad c^2 - s^2 = 1$

$$\frac{\partial y}{\partial x} = \sinh\left(\frac{1}{2}x\right)$$

$$\text{Surface Area} = 2\pi \int_{-\ln 2}^{\ln 2} 2\cosh\left(\frac{x}{2}\right)\sqrt{1 + \sinh^2\left(\frac{x}{2}\right)}\ \partial x$$

$$= 4\pi \int_{-\ln 2}^{\ln 2} \cosh^2\left(\frac{x}{2}\right)\partial x$$

$\cosh 2x = c^2 + s^2$

$= c^2 + c^2 - 1$

$\cosh 2x = 2c^2 - 1$

$$\cosh^2 x = \frac{(\cosh 2x + 1)}{2}$$

$$= \frac{4\pi}{2}\int_{-\ln 2}^{\ln 2} \cosh x + 1\ \partial x$$

$$\therefore \cosh^2\left(\frac{x}{2}\right) = \frac{(\cosh x + 1)}{2}$$

$$= 2\pi\left[\sinh x + x\right]_{-\ln 2}^{\ln 2}$$

$$= 2\pi\left(\frac{e^{\ln 2} - e^{-\ln 2}}{2} + \ln 2 - \frac{e^{-\ln 2} - e^{\ln 2}}{2} + \ln 2\right)$$

$$\cancel{= 2\pi\left(\frac{3}{4} + 2\ln 2 - \frac{3}{4}\right)} \qquad = 2\pi\left(\frac{3}{2} + 2\ln 2\right)$$

$$\cancel{= 4\pi\ln 2 \text{ units}^2} \qquad = 3\pi + 4\pi\ln 2$$

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3. $x = \dfrac{3}{\sinh\theta} = 3(\sinh\theta)^{-1}$

$$\frac{\partial x}{\partial \theta} = -3(\sinh\theta)^{-2}\cosh\theta$$

$$= -\frac{3\cosh\theta}{\sinh^2\theta}$$

$$\therefore \quad \partial x = -\frac{3\cosh\theta}{\sinh^2\theta}\,\partial\theta$$

$$\frac{1}{x\sqrt{x^2+9}} \equiv \frac{1}{\frac{3}{\sinh\theta}\sqrt{\frac{9}{\sinh^2\theta}+9}}$$

$$= \frac{1}{\frac{9}{\sinh\theta}\sqrt{\frac{1}{\sinh^2\theta}+1}}$$

$$\therefore \quad \frac{1}{x\sqrt{x^2+9}} \equiv \frac{\sinh\theta}{9\sqrt{\frac{1}{\sinh^2\theta}+1}}$$

$$\therefore \quad 3\sqrt{3} = \frac{3}{\sinh\theta} \Rightarrow \sinh\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cancel{\theta = \sqrt{3}} \quad \theta = \ln\sqrt{3}$$

$$4 = \frac{3}{\sinh\theta} \Rightarrow \sinh\theta = \frac{3}{4} \Rightarrow \theta = \ln 2$$

$$\therefore \int_{\ln 2}^{\ln\sqrt{3}} \frac{\sinh\theta}{9\sqrt{\frac{1}{\sinh^2\theta}+1}} \cdot -\frac{3\cosh\theta}{\sinh^2\theta}\,\partial\theta$$

$$= -\int_{\ln 2}^{\ln\sqrt{3}} \frac{3\cosh\theta}{9\sinh\theta\sqrt{\frac{1}{\sinh^2\theta}+1}}\,\partial\theta$$

$$= -\int_{\ln 2}^{\ln\sqrt{3}} \frac{3\cosh\theta}{9\sqrt{1+\sinh^2\theta}}\,\partial\theta$$

$$= -\int_{\ln 2}^{\ln\sqrt{3}} \frac{3}{9}\frac{\cosh\theta}{\sqrt{\cosh^2\theta}}\,\partial\theta$$

$$= -\int_{\ln 2}^{\ln\sqrt{3}} \frac{1}{3}\,\partial\theta = -\left[\frac{1}{3}\theta\right]_{\ln 2}^{\ln\sqrt{3}}$$

$$= -\left(\frac{1}{3}\ln\sqrt{3} - \frac{1}{3}\ln 2\right)$$

$$= \frac{1}{3}\ln 2 - \frac{1}{3}\ln\sqrt{3} = \frac{1}{3}\ln\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{1}{3}\ln\left(\frac{2}{3}\sqrt{3}\right) = \frac{1}{3}\ln\sqrt{\frac{4}{9}\times 3}$$

$$= \frac{1}{3}\ln\sqrt{\frac{4}{3}} = \frac{1}{6}\ln\left(\frac{4}{3}\right)$$

4 (a) $y = \arctan\sqrt{\cancel{x}}$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\therefore \tan y = \sqrt{x}$

$\therefore \frac{\partial y}{\partial x}\sec^2 y = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\tan^2 y + 1 = x + 1$

$\therefore \frac{\partial y}{\partial x}\left(\tan^2 y + 1\right) = \frac{1}{2\sqrt{x}}$

$\therefore \frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}} \cdot \frac{1}{x+1}$

$\frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}\,(x+1)}$

$\therefore \left(\frac{\partial y}{\partial x}\right)_{x=1/4} = \frac{1}{2\sqrt{\frac{1}{4}}\left(\frac{1}{4}+1\right)} = \frac{4}{5}$

(b) $\frac{\partial y}{\partial x} = \left(2\sqrt{x}\,(x+1)\right)^{-1}$

$\frac{1}{2}x^{-1/2}$

$\therefore \frac{\partial^2 y}{\partial x^2} = -\left(2\sqrt{x}\,(x+1)\right)^{-2}\left[2\sqrt{x}\,(1) + (x+1)\left(2\cdot\frac{1}{2}x^{-1/2}\right)\right]$

$= -\left(2\sqrt{x}\,(x+1)\right)^{-2}\left[2\sqrt{x} + \frac{(x+1)}{\sqrt{x}}\right]$

$= -\frac{2\sqrt{x} + \frac{(x+1)}{\sqrt{x}}}{\left(2\sqrt{x}\,(x+1)\right)^2} = -\frac{3x+1}{4\sqrt{x}\cdot x\,(x+1)^2}$

$= \frac{3x+1}{4x^{3/2}(x+1)^2}$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{3x+1}{4x^{3/2}(x+1)^2}$$

$$\therefore \quad 2x(1+x)\frac{\partial^2 y}{\partial x^2} = -\frac{3x+1}{4x^{3/2}(x+1)^2} \cdot 2x(1+x)$$

$$= -\frac{2x(3x+1)}{4x^{3/2}(x+1)} = -\frac{2(3x+1)}{4\sqrt{x}\,(x+1)}$$

$$= -\frac{3x+1}{2\sqrt{x}(x+1)}$$

$$\cancel{1+3x} \quad \frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}(x+1)}$$

$$\therefore \quad (1+3x)\frac{\partial y}{\partial x} = \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$\therefore \quad 2x(1+x)\frac{\partial^2 y}{\partial x^2} + (1+3x)\frac{\partial y}{\partial x} = -\frac{2(3x+1)}{4\sqrt{x}(x+1)} + \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$= -\frac{(3x+1)}{2\sqrt{x}(x+1)} + \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$= \frac{-(3x+1)+1+3x}{2\sqrt{x}(x+1)} = \frac{0}{2\sqrt{x}(x+1)}$$

$$= 0$$

as required.

5

$$\cancel{(a).\quad I_n = \int_0^{\pi/2} \sin^n x \, \partial x}$$

$$\cancel{u = \sin^n x \qquad u' = n \sin^{n-1} x \cos x}$$

$$\cancel{\therefore \quad v' = 1 \qquad v = x}$$

5 (a).

$$I_n = \int_0^{\pi/2} \sin^n x \, \partial x = \int_0^{\pi/2} \sin^{n-1} x \sin x \, \partial x$$

$$u = \sin^{n-1} x \qquad u' = (n-1) \sin^{n-2} x \cos x$$

$$v' = \sin x \qquad v = -\cos x$$

$$\therefore I_n = \Big[-\cos x \sin^{n-1} x \Big]_0^{\pi/2} + (n-1) \int_0^{\pi/2} \sin^{n-2} x \cos^2 x \, \partial x$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x \, (1 - \sin^2 x) \, \partial x$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x - \sin^n x \, \partial x$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x \; - (n-1) \int_0^{\pi/2} \sin^n x \, \partial x$$

$$\therefore I_n = (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore I_n + (n-1) I_n = (n-1) I_{n-2}$$

$$\therefore n I_n = (n-1) I_{n-2}$$

$\therefore \quad I_n - 2 = \frac{n-1}{n} I_{n-2}$ as required.

(b) $\int_0^{\pi/2} x \sin^5 x \cos x \, dx$

Let $u = x \qquad u' = 1$

$v' = \cos x (\sin x)^5 \qquad v = \frac{\sin^6 x}{6}$

$\therefore \int_0^{\pi/2} x \sin^5 x \cos x \, dx = \left[\frac{x \sin^6 x}{6} \right]_0^{\pi/2} - \frac{1}{6} I_6$

$\frac{1 - \cos 2x}{2}$

$1 - 2s^2 = \cos 2x$

$c^2 - s^2 = \cos 2x$

$s^2 \quad \cancel{c^2 + c - 1}$
$2c^2 - 1$

$= \frac{\pi}{12} - \frac{1}{6} I_6$

$= \frac{\pi}{12} - \frac{1}{6} \cdot \frac{5}{6} I_4$

$= \frac{\pi}{12} - \frac{5}{36} \cdot \frac{3}{4} I_2 \quad \cancel{= \frac{\pi}{12} - \frac{5}{48} \int_0^{\pi/2} \frac{1 - \cos 2x}{2} dx}$

$= \frac{\pi}{12} - \frac{5}{48} \cdot \frac{1}{2} I_0 \quad \cancel{= \frac{\pi}{12} - \frac{5}{96} \left[x - \frac{1}{2} \sin 2x \right]_0^{\pi/2}}$

$\cancel{= \frac{\pi}{12} - \frac{5}{96} \left(\frac{\pi}{2} \right)}$

$= \frac{\pi}{12} - \frac{5}{96} \int_0^{\pi/2} 1 \, dx$

0.0759...

$= \frac{\pi}{12} - \frac{5}{96} [x]_0^{\pi/2} \quad = \frac{\pi}{12} - \frac{5}{96} \left(\frac{\pi}{2} \right)$

$= \frac{\pi}{12} - \frac{5}{96} \left(\frac{6\pi}{12} \right) \quad \cancel{= \frac{91}{96} \frac{\pi}{12}}$

$= \frac{11}{192} \pi \qquad \cancel{= \frac{91}{1152} \pi} \quad \cancel{11}$

6 (a). $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$

$\overrightarrow{QR} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$

$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{array}{cccccc} -3 & 4 & -4 & -3 & 4 & -4 \\ 2 & 2 & -2 & 2 & 2 & -2 \end{array}$

$= \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$

(b) $\underset{\sim}{n} = \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$

$\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{p} \cdot \underset{\sim}{n}$

$\therefore \; -14y - 14z = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix} = 42 - 14$

$\therefore \; 14y + 14z = -28$

$y + z = -2$

(c). $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 6$

$\underset{\sim}{r} \cdot \begin{pmatrix} 0 \\ -1/4 \\ -1/4 \end{pmatrix} = 28$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $x + y - z = 6$

$y + z = -2$

$\therefore \quad y = -2 - z$

$x + -2 - z - z = 6$

$x - 2 - 2z = 6$

$x = 8 + 2z$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8+2z \\ -2-z \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + z\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$z = \lambda$

$\Rightarrow \underset{\sim}{r} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\underset{\sim}{r} \times \underset{\sim}{b} = \underset{\sim}{a} \times \underset{\sim}{b}$

$\underset{\sim}{a} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \quad \underset{\sim}{b} = \begin{matrix} 2 \\ -1 \\ 1 \end{matrix}$

$\cancel{\therefore \; r \times \begin{pmatrix} 2 \\ -1 \end{pmatrix}}$

$\therefore \left(r - \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix}\right) \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$

7(a). $A x = \begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 18+3k \\ 12 \\ -8 \end{pmatrix}$

$\cancel{= \begin{pmatrix} 3(6+k) \\ 3(4) \\ \end{pmatrix}}$ $\qquad \cancel{= 6\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}}$

$\cancel{= 4\begin{pmatrix} \\ 3 \\ -2 \end{pmatrix}}$

$\cancel{= \begin{pmatrix} 3(6+k) \\ 4(3) \end{pmatrix}}$

$= \begin{pmatrix} 4\left(\frac{9}{2} + \frac{3}{4}k\right) \\ 4(3) \\ 4(-2) \end{pmatrix} = 4\begin{pmatrix} \frac{9}{2} + \frac{3}{4}k \\ 3 \\ -2 \end{pmatrix}$

$\Rightarrow \frac{9}{2} + \frac{3}{4}k = 9 \quad \Rightarrow k = \left(9 - \frac{9}{2}\right) \times \frac{4}{3}$

$\therefore k = 6$ as required.

(b) $\lambda = 4$ is an eigen value

$$A - \lambda I = \begin{pmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)(1-\lambda)^2 - 6(1-\lambda)$$

$$= (1-\lambda)\left((2-\lambda)(1-\lambda) - 6\right)$$

$$= (1-\lambda)\left(2 - 3\lambda + \lambda^2 - 6\right)$$

$$= (1-\lambda)\left(\lambda^2 - 3\lambda - 4\right)$$

$$= (1-\lambda)(\lambda - 4)(\lambda + 1)$$

$\therefore$ $\lambda = 1$, $\lambda = 4$ are eigen-values
$\lambda = -1$

(c) $AP = \begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix}$

$= \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$

$\therefore$ the line represented has equation

$\underset{\sim}{r} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$

$\Rightarrow \quad x = 8t - 4 = \cancel{2(2t-2)}$

$y = 2t - 2 \qquad \Rightarrow \quad y + 2 = 2t$

$z = 0 \qquad \cancel{\therefore 2y+4=}$

$4y + 8 = 8t$

$\therefore x = 8t - 4 = 4y + 8 - 4 = 4y + 4$

$\therefore \quad x = 4y + 4$

$\Rightarrow x - 4y - 4 = 0$

8(a). $\frac{x^2}{25} - \frac{y^2}{9} = 1$

$\frac{2}{25}x - \frac{2}{9}y\frac{\partial y}{\partial x} = 0$

$\Rightarrow$ ~~$\frac{\partial y}{\partial x} = \frac{1}{225}\frac{x}{y}$~~ $\frac{\partial y}{\partial x} = \frac{9}{25}\frac{x}{y}$

$\therefore \left(\frac{\partial y}{\partial x}\right)$ at P = $\frac{9}{25} \cdot \frac{5}{3} \frac{\sec u}{\tan u} = \frac{3}{5}\operatorname{cosec} u$

$\therefore \quad y - 3\tan u = \frac{3}{5}(x - 5\sec u)\operatorname{cosec} u$

$\therefore \quad 5y - 15\tan u = 3\operatorname{cosec} u(x - 5\sec u)$

$\therefore \quad 5y - 15\tan u = 3\operatorname{cosec}(u)\, x - 15\operatorname{cosec}(u)\sec(u)$

~~$\div \operatorname{cosec} u$~~ $\times \sin u$ $\quad 5y\sin u - \frac{15\sin^2 u}{\cos u} = 3x - \frac{15}{\cos u}$

$\therefore$ ~~$5y\sin u$~~ $3x = 5y\sin u - \frac{15\sin^2 u}{\cos u} + \frac{15}{\cos u}$

$\therefore \quad 3x = 5y\sin u + \frac{15(1 - \sin^2 u)}{\cos u}$

since $1 - \sin^2 u = \cos^2 u$

$\therefore \quad 3x = 5y\sin u + 15\cos u$ as required

(b) at R $y = \frac{3}{5}x$

$\Rightarrow 3x = 5\left(\frac{3}{5}x\right)\sin u + 15\cos u$

$\therefore 3x = 3x\sin u + 15\cos u$

$\therefore \cancel{x = x}$ $(1-\sin u)(3x) = 15\cos u$

$\therefore x = \frac{5\cos u}{1-\sin u}$ & $y = \frac{3}{5}\left(\frac{5\cos u}{1-\sin u}\right)$

$= \frac{3\cos u}{1-\sin u}$

$\therefore R\left(\frac{5\cos u}{1-\sin u}, \frac{3\cos u}{1-\sin u}\right)$

at S, $y = -\frac{3}{5}x$

$\therefore \cancel{3y}$ $3x = 5\left(-\frac{3}{5}x\right)\sin u + 15\cos u$

$\therefore 3x = -3x\sin u + 15\cos u$

$\cancel{y = -\frac{3}{5}\left(\frac{5\cos u}{1+\sin u}\right.}$

$\therefore (1+\sin u)3x = 15\cos u$

$\therefore x = \frac{5\cos u}{1+\sin u}$ $y = \frac{-3\cos u}{1+\sin u}$

$\therefore$ at $S\left(\frac{5\cos u}{1+\sin u}, \frac{-3\cos u}{1+\sin u}\right)$

$\therefore$ x coordinate of

$$x_P = \frac{1}{2}\left(\frac{5\cos u}{1+\sin u} + \frac{5\cos u}{1-\sin u}\right)$$

$$x_P = \frac{1}{2}\left(\frac{5\cos u(1-\sin u) + 5\cos u(1+\sin u)}{1-\sin^2 u}\right)$$

$$\therefore x_P = \frac{1}{2}\left(\frac{5\cos u(1-\sin u) + 5\cos u(1+\sin u)}{\cos^2 u}\right)$$

$$\therefore x_P = \frac{1}{2}\left(\frac{5\cos u(2)}{\cos^2 u}\right) = \frac{5\cos u}{\cos^2 u} = \frac{5}{\cos u}$$

$$\therefore x_P = \frac{5}{\cos u} = 5\sec u$$

(b) at R $y = \frac{3}{5}x$

$\Rightarrow 3x = 5\left(\frac{3}{5}x\right)\sin u + 15\cos u$

$\therefore 3x = 3x\sin u + 15\cos u$

$\therefore \cancel{x = x}$ $(1-\sin u)(3x) = 15\cos u$

$\therefore x = \dfrac{5\cos u}{1-\sin u}$ & $y = \frac{3}{5}\left(\dfrac{5\cos u}{1-\sin u}\right)$

$= \dfrac{3\cos u}{1-\sin u}$

$\therefore R\left(\dfrac{5\cos u}{1-\sin u}, \dfrac{3\cos u}{1-\sin u}\right)$

at S, $y = -\frac{3}{5}x$

$\therefore \cancel{3y}$ $3x = 5\left(-\frac{3}{5}x\right)\sin u + 15\cos u$

$\therefore 3x = -3x\sin u + 15\cos u$ $\cancel{y = -\frac{3}{5}\left(\frac{5\cos u}{1+\sin u}\right.}$

$\therefore (1+\sin u)3x = 15\cos u$

$\therefore x = \dfrac{5\cos u}{1+\sin u}$ $y = \dfrac{-3\cos u}{1+\sin u}$

$$y_P = \frac{1}{2}\left(\frac{3\cos u}{1-\sin u} - \frac{3\cos u}{1+\sin u}\right)$$

$$= \frac{1}{2}\left(\frac{3\cos u(1+\sin u)}{1-\sin^2 u} - \frac{3\cos u(1-\sin u)}{1-\sin^2 u}\right)$$

$$= \frac{1}{2}\left(\frac{3\cos u(1+\sin u-(1-\sin u))}{\cos^2 u}\right)$$

$$= \frac{1}{2}\left(\frac{3(2\sin u)}{\cos u}\right) = \frac{3\sin u}{\cos u}$$

$$= 3\tan u$$ as required

$x_P = 5\sec u \qquad y_P = 3\tan u$

$\therefore$ P is midpoint of RS